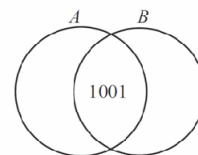


Combinatorics: The Breakfast of Mathletes**ICTM 2010 Annual Meeting and Conference — Friday, October 15, 2010****Sendhil Revuluri, University of Illinois at Chicago (sendhil@gmail.com)**

1. The pages of a book are consecutively numbered from 1 through 384. How many times does the digit 8 appear in this numbering? (*MOEMS Division M, 2008–2009 Contest 4, Problem 4D*)
2. The only way that 10 can be written as the sum of 4 different counting numbers is $1 + 2 + 3 + 4$. In how many different ways can 15 be written as the sum of 4 different counting numbers? (*MOEMS Division E, 2008–2009 Contest 4, Problem 4B*)
3. The number 632 is a multiple number since the first digit 6 is the product of the other two digits of the number. How many three-digit multiple numbers less than 600 exist? (*adapted from Saab MathCounts site*)
4. A haunted house has six windows. In how many ways can Georgie the Ghost enter the house by one window and leave by a different window? (*2007 AMC 8, Problem 4*)
5. Sets A and B , shown in the Venn diagram, have the same number of elements. Their union has 2007 elements and their intersection has 1001 elements. Find the number of elements in A . (*2007 AMC 8, Problem 13*)
6. A deli offers 5 different salads, 6 different drinks, and 7 different pastas. How many combinations of 2 appetizers, 2 drinks and 2 pastas could someone choose for a picnic? (*adapted from Saab MathCounts site*)
7. How many even four-digit numbers greater than 7000 can be formed using the digits from the set $\{3,4,5,7,9\}$ when no repetition of digits is allowed in a number? (*adapted from Saab MathCounts site*)
 - a. How many 4-digit numbers can be formed using the digits from the set $\{3,4,5,7,9\}$?
 - b. How many 4-digit numbers can be formed from those digits when no repetition of digits is allowed?
 - c. How many 4-digit numbers can be formed from those digits that are greater than 7000?
8. Two boys and two girls are officers of the Math Club. When the photographer takes a picture for the school yearbook, she asks the clubs four officers and the one faculty sponsor to sit in a row with the faculty sponsor in the middle and the two boys not next to one another. How many different seating arrangements are possible? (*adapted from Saab MathCounts site*)



9. How many orders are there of the letters in the word “ILLINOIS”? In the word “MASSACHUSETTS”? (adapted from “The Amazing Race”, problem series on the Art of Problem Solving discussion forums)
10. Bernardo randomly picks 3 distinct numbers from the set $\{1,2,3,4,5,6,7,8,9\}$ and arranges them in descending order to form a 3-digit number. Silvia randomly picks 3 distinct numbers from the set $\{1,2,3,4,5,6,7,8\}$ and also arranges them in descending order to form a 3-digit number. What is the probability that Bernardo's number is larger than Silvia's number? (2010 AMC 10A, Problem 18)
11. Two cards are dealt from a deck of four red cards labeled A, B, C, D and four green cards labeled A, B, C, D . A winning pair is two of the same color or two of the same letter. What is the probability of drawing a winning pair? (2007 AMC 8 Problem 21)
12. A bag contains four pieces of paper, each labeled with one of the digits “1”, “2”, “3” or “4”, with no repeats. Three of these pieces are drawn, one at a time without replacement, to construct a three-digit number. What is the probability that the three-digit number is a multiple of 3? (2007 AMC 8, Problem 24)
13. The product of 180 and the positive integer N is a perfect cubic number. What is the least possible value of N ? (MOEMS Division M, 2008–2009 Contest 4, Problem 4E)
14. How many positive numbers that go into 6,528 are even? (from “The Amazing Race”, problem series on the Art of Problem Solving discussion forums)
15. If you have eight identical candies, how many ways can you:
- Put them into two piles?
 - Give them to two kids, Avery and Bree?
 - What if the candies weren't identical? (Are there more or fewer ways? Is this easier or harder?)
16. How many four-digit numbers between 8000 and 9000 are there for which the thousands digits equal the sum of the other three digits? (adapted from Saab MathCounts site)
17. The polynomial $x^3 - ax^2 + bx - 2010$ has three positive integer zeros. What is the smallest possible value of a ? (2010 AMC 10A, Problem 21)
18. Eight points are chosen on a circle, and chords are drawn connecting every pair of points. No three chords intersect in a single point inside the circle. How many triangles with all three vertices in the interior of the circle are created? (2010 AMC 10A, Problem 22)