While you're waiting:

Consider the following problem:

> Find an algebraic rule to describe the relationship between the number of sides of a polygon, ***n***, and the number of diagonals that can be drawn from one vertex, ***d***.
>
> Use ***n*** as the independent variable, and express your rule using function notation.
>
> Explain how you found the rule.

- Work the problem. (You may consult a neighbor.)
- Then, think about your students.
- What are some challenges 9th-grade Algebra 1 students would face in solving a problem like this?

Reclaiming Lost Ground: Research-based Interventions for Under-prepared Algebra Students

Sendhil Revuluri, University of Illinois at Chicago
Stephen Spring, Dana Center, University of Texas

NCTM Regional Conference and Exposition
New Orleans — Thursday, October 28, 2010

FYI

Presentation slides will be posted on the Intensified Algebra website:

utdanacenter.org/intensifiedalgebra/index.php

Why are we here?

Algebra is now the reason for more student dropouts than any other subject. The frustration of consistently failing algebra creates a sense of futility. As a result, more and more students are giving up the hope of a high school diploma from the Los Angeles schools… ***We need an approach to teaching algebra that helps these students succeed****.*

— Roy Romer, former Superintendent, Los Angeles Unified School District

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The audience and compelling need

Students who struggle most with algebra not only have significant mathematical deficiencies, many are also turned off and ***alienated*** *from school, have had little academic and intellectual* ***success****, and express a sense of* ***hopelessness*** *about completing high school.*

[They need] a program of ***excellent quality*** *with* ***high mathematics expectations****. They also need a program that captures their spirit and inspires them to believe in* ***their ability to succeed*** *in mathematics.*

— Richard Kaplan, Algebra Teacher

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Session Goals

- To provide information about research-informed strategies and resources to help under-prepared algebra students succeed.
- To illustrate how you might incorporate these strategies into your instruction.

Intensified Algebra

- NSF funded research and development initiative
- Design-based research approach to curriculum development
- Grew out of need identified by the Urban Mathematics Leadership Network (UMLN)

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Opening question

Find an algebraic rule to describe the relationship between the number of sides of a polygon, ***n***, and the number of diagonals that can be drawn from one vertex, ***d***.

Use ***n*** as the independent variable, and express your rule using function notation.

Explain how you found the rule.

What are some challenges 9th-grade students would face in solving a problem like the diagonals problem, mathematical or otherwise?

What types of scaffolds, interventions, or classroom norms would help support student success with a problem like this?

The Need

- It's essential for all 9th graders to succeed in Algebra 1, even those who are underprepared for high school mathematics.

- The predominant strategy — increasing instructional time through double-period courses — has had only limited success.

Research provides guidance about supporting under-prepared students…

- Mathematics education
- Cognitive science
- Social psychology
- Special education and struggling learners

How can we translate research results into instructional strategies to support algebra students?

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Based on research about…

- How people learn
- How students learn mathematics
- Particular challenges in learning algebra
- New approaches to knowing what students know
- Established principles of mathematics learning and instruction
- Needs of struggling adolescent learners
- Socio-emotional aspects of academic success

Research tells us learners should…

- Engage with challenging tasks that involve active meaning-making.
- Connect new learning with prior knowledge and, in the process, address misconceptions.
- Acquire conceptual knowledge as well as skills to enable them to organize their knowledge, transfer knowledge to new situations, and acquire new knowledge.
- Socially construct knowledge through talk, activity, and interaction around meaningful problems.

Research tells us learners should…

- Receive timely feedback so they can revise their work, thinking, and understandings.
- Employ metacognitive awareness of their performance.
- Practice the skills, concepts, and principles they acquire in tasks and situations that are close to the original learning situation as well as more distant from it.
- Understand that intelligence is malleable, not fixed, and recognize the roles of effective effort, attribution of effort, sense of belonging, and motivation in learning mathematics.

Struggling Learners Need

- Routines and structures to help them access and organize critical mathematics content.

- Efficient, timely opportunities to surface and repair misconceptions — in ways that don't delay access to new mathematics learning.

The Power of Routines

"Some students in school today don't see a connection between their efforts and school success, don't know what it is they need to practice, can't imagine themselves ever being 'academic,' and have never seen 'academics played.' . . .

A first step in helping students become full participants in the classroom is to ensure that all students value and understand the importance of learning and **learning rituals**."

Lenz, B.K., Deshler, D. (2004). Teaching Content to All: Evidence-Based Practices in Middle and Secondary Schools. Boston: Pearson Education, Inc.

Core Idea: Be Explicit

- Communicate clear expectations and help students structure content.
- Structure experiences to make students' thinking visible.
- Structure experiences to promote students' taking responsibility for their own learning.

Help students organize content: Lesson preview

In this lesson, you will focus on the topic goals that are highlighted below.

- **Use words, tables, graphs, and algebraic rules to identify, describe, and analyze patterns and mathematical relationships**
- **Solve problems and model real-world situations using patterns and mathematical relationships**
- **Make connections among representations of mathematical relationships, using verbal descriptions, tables, graphs, and algebraic rules**
- Determine the advantages and limitations of using a particular representation to answer a question
- Analyze and create equivalent algebraic expressions and rules
- Demonstrate an introductory understanding of "allowable inputs," discrete and continuous data, and proportional and non-proportional linear relationships

Intensified Algebra, Topic 4

Help students organize content: Lesson preview

The activities for today's lesson are described below.

2.1 Opener: Generalizing a dot pattern
Find an algebraic rule for a pictorial pattern

2.2 The Banquet Table Problem
Use multiple representations to describe a mathematical relationship for a problem with a real-life context

2.3 Process homework

2.4 Review end-of-unit assessment
Review Unit 1 end-of-unit assessment

2.5 Wrap up and introduce homework

Intensified Algebra, Topic 4

Help students organize content: Lesson wrap-up

Activity 2.5 **Wrap up and introduce homework**

2.1	**Opener: Generalizing a dot pattern** Find an algebraic rule for a pictorial pattern
2.2	**The Banquet Table Problem** Use multiple representations to describe a mathematical relationship for a problem with a real-life context
2.3	**Process homework**
2.4	**Review end-of-unit assessment** Review Unit 1 end-of-unit assessment
2.5	**Wrap up and introduce homework**

Reflect on today's lesson and be prepared to share out with the class:

An important idea from today's lesson is ...

Look at tonight's homework:

Which activities from today's lesson prepare you to successfully complete the homework?

Intensified Algebra, Topic 4

Connect New Learning with Prior Knowledge

A newspaper is better than a magazine, and on a seashore is a better place than a street. At first it is better to run than walk. Also you may have to try several times. It takes some skill but it's very easy to learn. Even young children can enjoy it. Once successful, complications are minimal. Birds seldom get too close. One needs lots of room. Rain soaks in very fast. Too many people doing the same thing can also cause problems. If there are no complications, it can be very peaceful. A rock will serve as an anchor. If things break loose from it, however, you will not get a second chance.

Connect New Learning with Prior Knowledge

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Openers

ACTIVITY 2.1 Opener: Generalizing a dot pattern

A sequence is shown in picture form. Assume that the sequence continues to grow in the same way. Study the pattern. Then complete the following tasks.

Figure 1 Figure 2 Figure 3 Figure 4

1. Organize the information for the figures into an input-output table.

Figure number, n	Number of dots, d

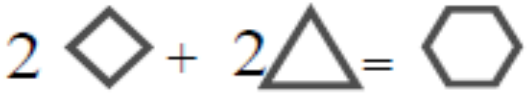
2. Write an input-output rule for the table. Your rule should use the variables n and d.

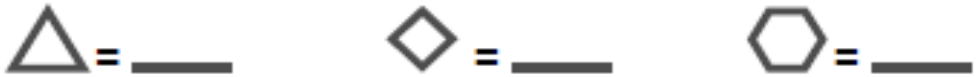
Reflection

- Are you using openers/warm-ups?
- What function is your opener/warm-up serving?
- What other opportunities do you have to help students access prior knowledge?

Day 1 Activity

Shape Equations #5

Clue 1 	**Clue 2** 
Clue 3 	**Clue 4** 



Your thinking:

Intensified Algebra,
Topic 1

Substitution Method for Solving Systems

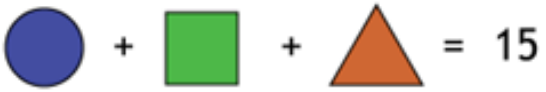
Intensified Algebra, Topic 18

Activity 12.1 Opener: Return of the shape equations 1-2

Work on the following problems in your activity book.

1. Evaluate $ab + 2c$ when $a = -2$, $b = 3$, and $c = 5$.

2. Following is a set of three shape equations. The value for each shape is constant in the three equations. Find the values for the shapes. Then explain your reasoning.



Build Upon Informal Knowledge

1. Mike has 8 pennies. Sam has 3 pennies. How many altogether?
2. Mike has 8 pennies. Sam gives him 3 more. How many does Mike have now?
3. Mike has 8 pennies. He loses 3. How many does he have now?
4. Mike has 8 pennies. Sam gives him some more. Now he has 11. How many did he get from Sam?
5. Mike has 11 pennies. He loses some. How he has 8 pennies. How many did he lose?
6. Mike has some pennies. He gets 3 more. Now he has 11. How many did he have at the beginning?
7. Mike has some pennies. He loses 3. Now he has 8. How many did he have at the beginning.

Build Upon Informal Knowledge

Carla's Cars-to-Go charges a $50 flat fee plus $0.25 per mile to rent an van.

- What is the cost of the rental if you drive 40 miles?
- What is the cost of the rental if you drive 100 miles?
- What is the cost of the rental if you drive 200 miles?
- A rental costs $110. How many miles were driven?

$y = 0.25x + 50$

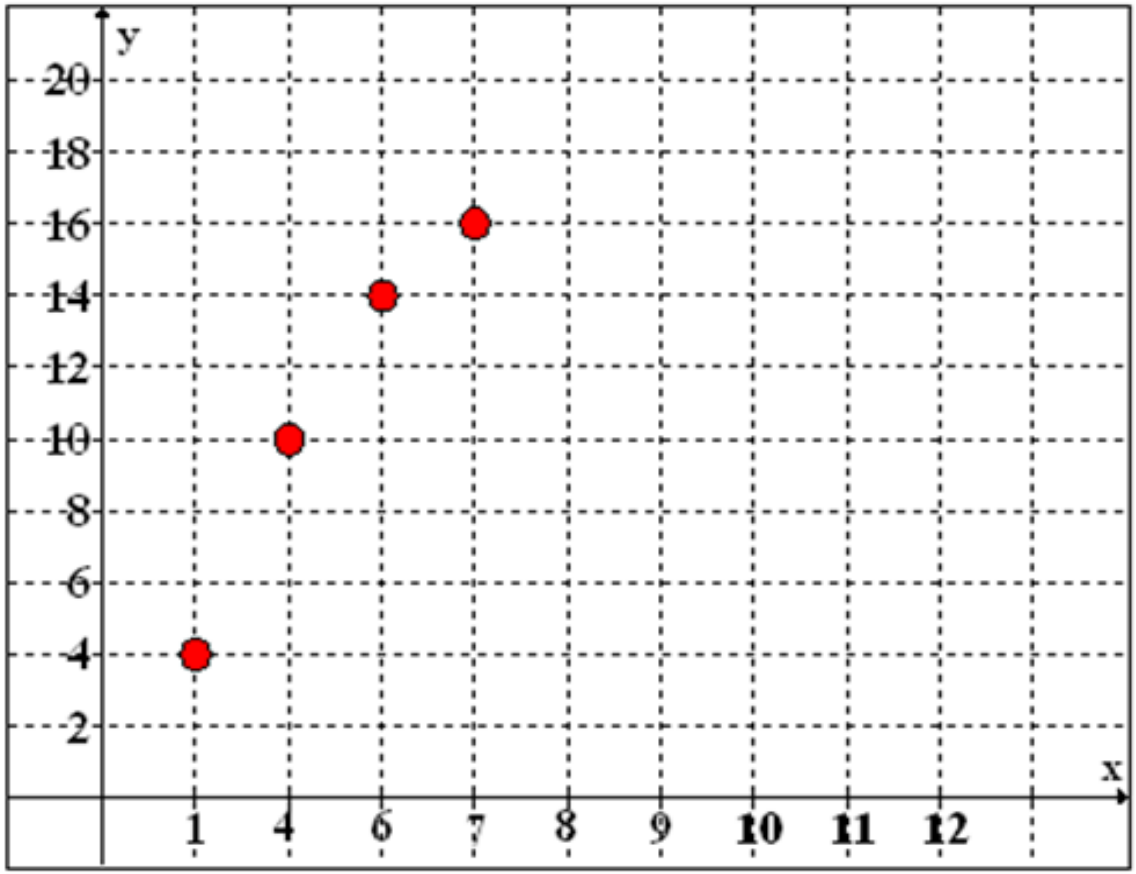
- Solve for y when x = 40, 100, 200.
- Solve $110 = 0.25x + 50$

Surfacing and Repairing Misconceptions

Table 1

x	y
1	4
4	10
6	14
7	16

Graph 1



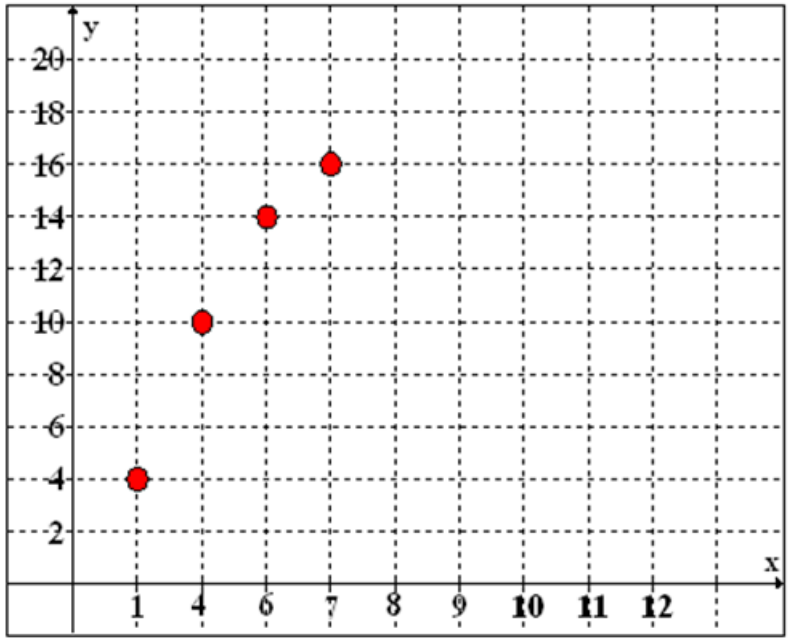
Analyzing worked examples

1. What's wrong with Graph 1?

Table 1

x	y
1	4
4	10
6	14
7	16

Graph 1



2. Use the data in Table 1 to draw a correct graph.

Students aren't blank slates — so…

- Access prior knowledge
- Build on informal knowledge
- Surface and repair misconceptions

Learners should

- Engage with challenging tasks that involve active meaning-making.
- Acquire conceptual knowledge as well as skills to enable them to organize their knowledge, transfer knowledge to new situations, and acquire new knowledge.

Hiebert & Grouws, 2007

What Are Mathematical Tasks?

Mathematical tasks are a set of problems, or a single complex problem, the purpose of which is to focus students' attention on a particular mathematical idea.

Why Focus on Mathematical Tasks?

- Tasks form the basis for students' opportunities to learn what mathematics is and how one does it;
- Tasks influence learners by directing their attention to particular aspects of content and by specifying ways to process information;
- The level and kind of thinking required by mathematical instructional tasks influences what students learn; and
- Differences in the level and kind of thinking of tasks used by different teachers, schools, and districts, is a major source of inequity in students' opportunities to learn mathematics.

What level and kind of thinking is called for?

Martha was re-carpeting her bedroom which was 15 feet long and 10 feet wide. How many square feet of carpeting will she need to purchase?

Stein, Smith, Henningsen, & Silver, 2000, p. 1

Raising the Cognitive Demand

Ms. Brown's class will raise rabbits for their spring science fair. They have 24 feet of fencing with which to build a rectangular rabbit pen in which to keep the rabbits.

1. **If Ms. Brown's students want their rabbits to have as much room as possible, how long would each of the sides of the pen be?**
2. **How long would each of the sides of the pen be if they had only 16 feet of fencing?**
3. **How would you go about determining the pen with the most room for any amount of fencing? Organize your work so that someone else who reads it will understand it.**

Stein, Smith, Henningsen, & Silver, 2000, p. 2

The Banquet Table Problem

Erlinda and Chris continue their work on the dance committee. Erlinda just found out the hall where the tables will be located is long and narrow. There is not enough room to spread the tables out. Chris suggests pushing the tables together in a row. He makes diagrams showing arrangements of tables and chairs.

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The Banquet Table Problem

1. How many people can sit around 4 hexagonal tables pushed together? 5 tables?
2. Determine how many people can sit around 10 tables pushed together without drawing a diagram.
3. Find a general algebraic rule for the number of people that can sit around any number of hexagonal tables that are pushed together. Write your rule using the variables ***n*** and ***p***.

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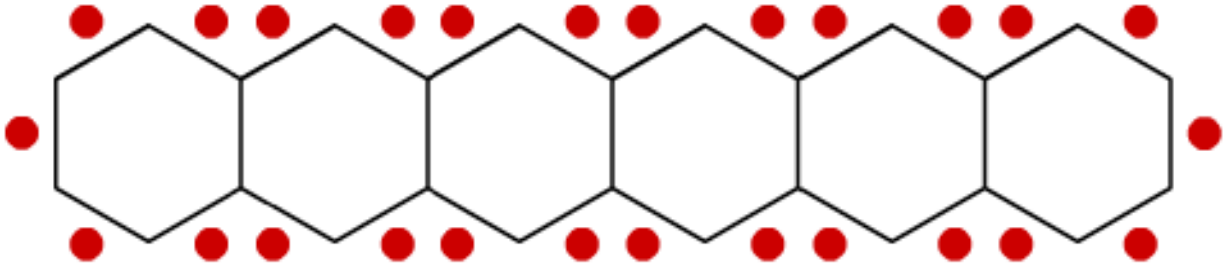
Extending the Banquet Table Problem



Explain what each person was thinking to find the number of people who could sit around 6 tables pushed together. Connect your explanation to the picture of the tables.

Caterer:	$1 + (2 \cdot 12) + 1 = 26$
Erlinda:	$5 + (4 \cdot 4) + 5 = 26$
Chris:	$1 + (4 \cdot 6) + 1 = 26$
Pauline:	$6 \cdot 6 - (5 \cdot 2) = 26$

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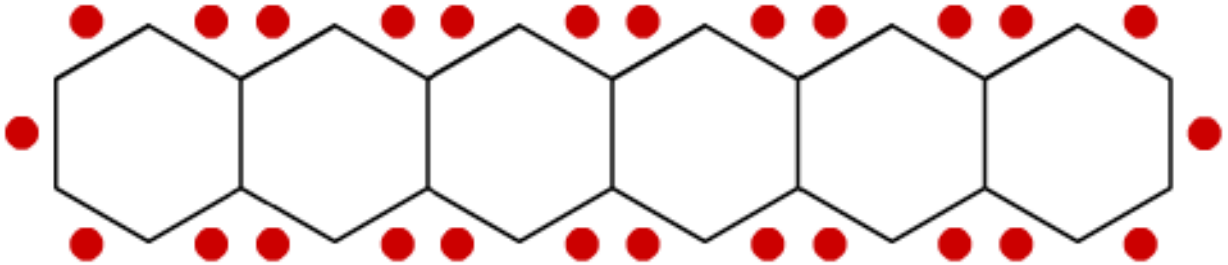
Research Results

- **Combine graphics with verbal descriptions**
  to facilitate encoding of individual mathematical representations and to make conceptual connections between representations.
- **Incorporate analyzing and explaining examples of both correct and incorrect solutions**
  Incorrect examples that anticipate common student misconceptions push students to more deeply process and reason with greater understanding.

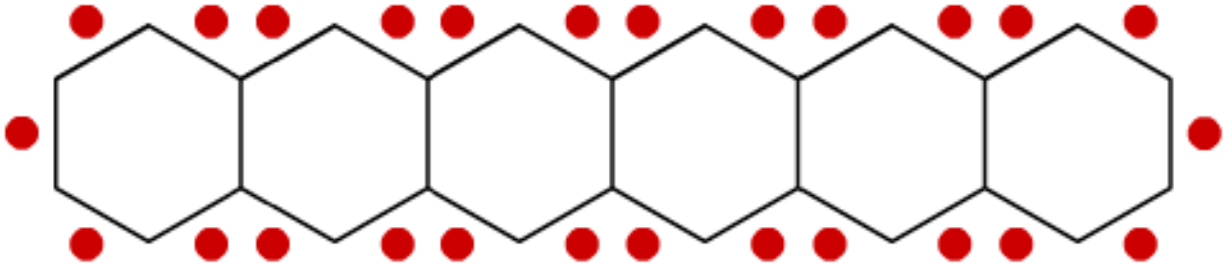
IES Practice Guide, 2007

Extending the Banquet Table Problem



- Show how each person would calculate the number of people that can be seated around 9 tables.
- Generalize from the arithmetic to write an algebraic rule that gives the number of people that can be seated around any number of tables. These rules should link the number of tables (***n***) to the number of people (***p***) that can be seated.

Extending the Banquet Table Problem



Rewrite each rule more simply:

Caterer: $p = 1 + 2(2n) + 1$

Erlinda: $p = 5 + 4(n - 2) + 5$

Chris: $p = 1 + 4n + 1$

Pauline: $p = 6n - 2(n - 1)$

Which rules are equivalent?

Core Structure: Partner Work

- Engages all students throughout the lesson.
- Gives students fewer places to hide.
- Partner talk and partner work makes students' thinking visible.

Research Result

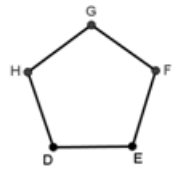
Providing students with periodic opportunities to practice using concepts and skills, along with feedback about their performance, helps students solidify their knowledge and promotes retention, reflection, generalization, and transfer of knowledge and skill.

IES Practice Guide, 2007

Routine: Distributed practice

Staying Sharp 12

1. Polygon DEFGH is a regular pentagon. If the sum of all of the interior angles is 540°, what is the measure of $\angle E$?



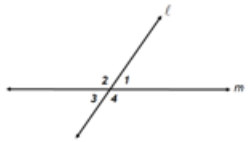
Answer with supporting work:

2. In the diagram shown below, line ℓ intersects line m. If the measure of $\angle 1$ is 70°, find:

$m\angle 2 =$ ____

$m\angle 3 =$ ____

$m\angle 4 =$ ____



3. The following points are on the graph of a particular line: $(-2,6)$, $(0,5)$, $(2,4)$, $(6,2)$, $(8,1)$, $(10,0)$, $(12,-1)$. What is the y-intercept of the line?

Answer with supporting work:

4. Complete four rows of an x-y table for the following function: $y = 2x + 7$

x	y

5. Bobby and Billy sit on one side of a seesaw. When their older brother, Adam, who weighs 110 pounds, sits on the other side, the seesaw tilts down on Adam's side. What can you conclude about Bobby and Billy's combined weight? What can you conclude about their individual weights?

Answer with explanation

6. Adam sits on one side of a seesaw; his twin brothers sit on the other side; the seesaw is perfectly balanced. Then, the twins sit on one side and Mr. Howard and his dog sit on the other side; the scale again is perfectly balanced. What will happen if Adam sits on one side of the seesaw and Mr. Howard and his dog sit on the other?

Answer with explanation:

What about test-prep?

Too often, teachers are putting regular instruction "on hold" to spend class time practicing test questions. While on the surface this may appear to make sense, research indicates just the opposite—

test scores are actually lower in schools where teachers spend large amounts of time on test prep.

(Allensworth, Correa, & Ponisciak, 2008)

Research Result

- Promoting students' metacognitive awareness and taking responsibility for their own learning increases student achievement.

Students' Beliefs about their Intelligence Affect their Academic Achievement

- Fixed mindset:
  - Avoid learning situations if they might make mistakes
  - Try to hide, rather than fix, mistakes or deficiencies
  - Decrease effort when confronted with challenge

- Growth mindset:
  - Work to correct mistakes and deficiencies
  - View effort as positive; increase effort when challenged

Dweck, 2007

Routine: Processing homework

◀ 11-The substitution method ▶ ◀ 1 2 3 4 5 6 7 ▶

ACTIVITY 11.4 **Process homework**

WITH YOUR PARTNER:

- Compare your answers
- Compare how you solved the problem.

ON YOUR OWN:

Green = I understand all of the ideas in the homework.

Yellow = I understand some/most of the ideas.

Red = I don't understand most of the ideas in the homework.

- Place your corrected homework assignment in the IN side of the folder.
- Remove any assignments on the OUT side of the folder.

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Students' Beliefs about their Intelligence Affect their Academic Achievement

When confronted with challenging school transitions or courses, students with growth mindsets outperform those with fixed mindsets, even when they enter with equal skills and knowledge.

Dweck, 2007

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Students Can Develop Growth Mindsets

- Explicit instruction about the brain, its function, and that intellectual development is the result of effort and learning has increased students' achievement in middle school mathematics.
- Teacher praise influences mindsets
  - Fixed: Praise refers to intelligence
  - Growth: Praise refers to effort, engagement, perseverance

How Research Can Inform Our Practice

- Providing routines and structures that help struggling learners organize critical mathematics content increases their learning.
- Accessing prior knowledge and addressing students' misconceptions increases learning.
- Engaging students with challenging tasks that involve active meaning-making increases learning.
- On-going, cumulative, distributed practice improves learning and retention.
- Developing a growth mindset can increase learners' motivation and effort to learn mathematics.

Thank You!

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